General Certificate of Education
Advanced Level Examination
June 2013

# Mathematics MM2B

## Unit Mechanics 2B

## Thursday 13 June 2013 9.00 am to 10.30 am

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**1** A particle, of mass 3 kg, moves along a straight line. At time $t$ seconds, the displacement, $s$ metres, of the particle from the origin is given by

$$s = 8t^3 + 15$$

**(a)** Find the velocity of the particle at time $t$. *(2 marks)*

**(b)** Find the magnitude of the resultant force acting on the particle when $t = 2$. *(4 marks)*

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**2** Carol, a circus performer, is on a swing. She jumps off the swing and lands in a safety net. When Carol leaves the swing, she has a speed of $7\,\text{m}\,\text{s}^{-1}$ and she is at a height of 8 metres above the safety net.

Carol is to be modelled as a particle of mass 52 kg being acted upon only by gravity.

**(a)** Find the kinetic energy of Carol when she leaves the swing. *(2 marks)*

**(b)** Show that the kinetic energy of Carol when she hits the net is 5350 J, correct to three significant figures. *(3 marks)*

**(c)** Find the speed of Carol when she hits the net. *(3 marks)*

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**3** A particle, of mass 10 kg, moves on a smooth horizontal plane. At time $t$ seconds, the acceleration of the particle is given by

$$\{(40t + 3t^2)\,\mathbf{i} + 20\text{e}^{-4t}\,\mathbf{j}\}\,\text{m}\,\text{s}^{-2}$$

where the vectors **i** and **j** are perpendicular unit vectors.

**(a)** At time $t = 1$, the velocity of the particle is $(6\mathbf{i} - 5\text{e}^{-4}\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

Find the velocity of the particle at time $t$. *(5 marks)*

**(b)** Calculate the initial speed of the particle. *(3 marks)*

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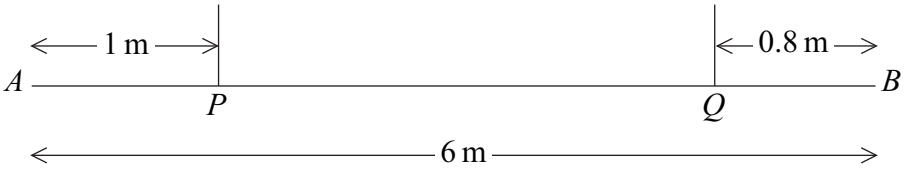
**4** A uniform plank $AB$, of length 6 m, has mass 25 kg. It is supported in equilibrium in a horizontal position by two vertical inextensible ropes. One of the ropes is attached to the plank at the point $P$ and the other rope is attached to the plank at the point $Q$, where $AP = 1\,\text{m}$ and $QB = 0.8\,\text{m}$, as shown in the diagram.




**(a) (i)** Find the tension in each rope. *(5 marks)*

**(ii)** State how you have used the fact that the plank is uniform in your solution. *(1 mark)*

**(b)** A particle of mass $m$ kg is attached to the plank at point $B$, and the tension in each rope is now the same.

Find $m$. *(6 marks)*

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**5** Tom is travelling on a train which is moving at a constant speed of $15\,\text{m s}^{-1}$ on a horizontal track. Tom has placed his mobile phone on a rough horizontal table. The coefficient of friction between the phone and the table is $0.2$.

The train moves round a bend of constant radius. The phone does not slide as the train travels round the bend.

Model the phone as a particle moving round part of a circle, with centre $O$ and radius $r$ metres.

Find the least possible value of $r$. *(4 marks)*

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**6** A car accelerates from rest along a straight horizontal road.

The car's engine produces a constant horizontal force of magnitude 4000 N.

At time $t$ seconds, the speed of the car is $v\,\text{m s}^{-1}$, and a resistance force of magnitude $40v$ newtons acts upon the car.

The mass of the car is 1600 kg.

**(a)** Show that $\dfrac{\text{d}v}{\text{d}t} = \dfrac{100 - v}{40}$. *(2 marks)*

**(b)** Find the velocity of the car at time $t$. *(6 marks)*

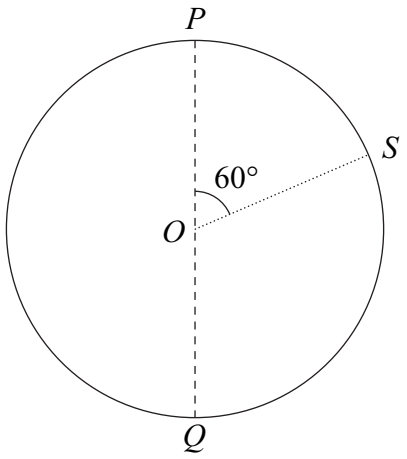
**7** A train, of mass 22 tonnes, moves along a straight horizontal track. A constant resistance force of 5000 N acts on the train. The power output of the engine of the train is 240 kW.

Find the acceleration of the train when its speed is $20\,\text{m s}^{-1}$. *(6 marks)*

**8** A bead, of mass $m$, moves on a smooth circular ring, of radius $a$ and centre $O$, which is fixed in a vertical plane. At $P$, the highest point on the ring, the speed of the bead is $2u$; at $Q$, the lowest point on the ring, the speed of the bead is $5u$.

**(a)** Show that $u = \sqrt{\frac{4ag}{21}}$. *(4 marks)*

**(b)** $S$ is a point on the ring so that angle $POS$ is 60°, as shown in the diagram.




Find, in terms of $m$ and $g$, the magnitude of the reaction of the ring on the bead when the bead is at $S$. *(5 marks)*

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**9** Two particles, $A$ and $B$, are connected by a light elastic string that passes through a hole at a point $O$ in a rough horizontal table. The edges of the hole are smooth. Particle $A$ has a mass of 8 kg and particle $B$ has a mass of 3 kg.

The elastic string has natural length 3 metres and modulus of elasticity 60 newtons.

Initially, particle $A$ is held 3.5 metres from the point $O$ on the surface of the table and particle $B$ is held at a point 2 metres vertically below $O$.

The coefficient of friction between the table and particle $A$ is 0.4.

The two particles are released from rest.

**(a) (i)** Show that initially particle $A$ moves towards the hole in the table. *(3 marks)*

**(ii)** Show that initially particle $B$ also moves towards the hole in the table. *(2 marks)*

**(b)** Calculate the **initial** elastic potential energy in the string. *(2 marks)*

**(c)** Particle $A$ comes permanently to rest when it has moved 0.46 metres, at which time particle $B$ is still moving upwards.

Calculate the distance that particle $B$ has moved when it is at rest for the first time. *(7 marks)*